Towards the Realization of Fractional Step Filters

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Outline

• Intro to Fractional Filters
• Passband Peaking Problem
• Proposed Fractional Transfer Function
• Stability
• Higher Order Implementation
• High Pass Fractional Filter
• Physical Realization
• Experimental Results

Towards the Realization of Fractional Step Filters
Intro to Fractional Filters

- What are fractional filters?
Fractional Calculus

- Riemann-Liouville definition
  \[ \frac{d^\alpha}{dt^\alpha} = D^\alpha f(t) = \frac{1}{\Gamma(1 - \alpha)} \frac{d}{dt} \int_0^t (t - \tau)^{-\alpha} f(\tau) d\tau \]

  where \( 0 < \alpha < 1 \)

- Fractional Laplacian Operator
  \[ L\{0 d_t^\alpha f(t)\} = s^\alpha F(s) \]

- A simple fractional filter would be
  \[ H^{1+\alpha}(s) = \frac{k_1}{s^{1+\alpha} + k_2} \]
Passband Peaking

- Occurs as the poles move closer to the region of instability

\[ H^{LP}_{1+\alpha}(s) = \frac{k_1}{s^{1+\alpha} + k_2} \]

Magnitude Response of Fractional Step Filter between 1st and 2nd orders
Proposed Transfer Function

\[ H_{1+\alpha}^{LP} = \frac{k_1}{s^\alpha(s + k_2) + k_3} \]

\[ = \frac{k_1}{s^{1+\alpha} + k_2 s^\alpha + k_3} \]

● Compared with

\[ H^{1+\alpha}(s) = \frac{k_1}{s^{1+\alpha} + k_2} \]
Simulations

\[ H_{1+\alpha}^{LP}(s) = \frac{k_1}{s^{1+\alpha} + k_2} = \frac{1}{s^{1+0.9} + 1} \]

\[ H_{1+\alpha}^{LP}(s) = \frac{k_1}{s^\alpha(s + k_2) + k_3} = \frac{1}{s^{0.9}(s + 1.31) + 0.99} \]

Slope
\[ = -20(1 + 0.9)dB/\text{dec} \]
\[ = -38dB/\text{dec} \]
k_{2,3} Selection

- Accomplished through selecting the combination of k_{2,3} that yield the least cumulative error compared to the 1st order Butterworth, with the cumulative error calculated as

$$|E_C(j\omega)| = \sum_{i=1}^{N} |B_1(j\omega_i)| - |H_{1+\alpha}^{LP}(j\omega_i)||$$

\[k_3 = 0.19295\alpha + 0.81369, \quad R = 0.024265\]

\[k_2 = 1.1796\alpha^2 + 0.16765\alpha + 0.21735, \quad R = 0.090685\]
Stability

- We define a W-plane such that
  \[ s = W^m \]
  \[ \alpha = \frac{k}{m} \]
- Characteristic equation becomes:
  \[ W^{m+k} + k_2 W^k + k_3 = 0 \]
- Solve the characteristic equation in the W-plane for the stability criteria
  \[ | \theta_W | > \frac{\pi}{2m} \]
Minimum Root Angles

$m = 100$

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Higher Order Implementation

- System is always stable for \((n + \alpha) < 2\)
- Always Unstable for \((n + \alpha) \geq 2, n \geq 2\)
- How do we implement higher order filters?

\[
H_{n+\alpha}^{LP} \approx \frac{H_{1+\alpha}^{LP}}{B_{n-1}(s)}
\]

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High Pass Fractional Filter

- We can still apply the LP-to-HP transformation
Approximation of $s^\alpha$

- Currently no commercial fractional capacitors exist, though some materials exhibit fractional properties

$\alpha = 0.5$

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Physical Realization

- \( H_{1+\alpha}^{LP} \approx \frac{k_1}{a_0} \frac{(a_2 s^2 + a_1 s + a_0)}{s^3 + c_0 s^2 + c_1 s + c_2} \)

- Realized using the STAR circuit and a bilinear block.
Experimental Results of $H_{1+\alpha}(s)$

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Conclusions

- Proposed a fractional step filter that removes the passband peaking of previous fractional step filters.
- Shown a method for implementing higher order and high-pass filters.
- Use of integer order approximations to $s^\alpha$ are appropriate to realize these filters, until fractal devices become available.
References


Questions?